Ramsaday College, Amta
Class Test 2018
Subject: Mathematics

B.Sc.(Hons.)-Ist year **Abstract Algebra**

F.M. 30 **Time : 1 Hour 15 Min.**

Answer any six questions:

1. Let A, B, C denote the subset of S and C' denotes the complement of C in S. If $A \cap C = B \cap C$ and $A \cup C' = B \cup C'$ then prove that $A = B$.
2. Let R be a relation defined on the set of integers Z such that $R = \{(a,b): a,b \in Z; a - b = 5n, n \in Z\}$. Show that R is an equivalence relation. If R' is another relation defined by $R' = \{(a,b): a,b \in Z; a - b = 3n, n \in Z\}$ then show that $R \cup R'$ is symmetric but not transitive.
3. What do you mean by a finite group? Let $(G,*)$ be a group. State and prove the necessary and sufficient condition(s) for a subset of a G be a subgroup of G. What will happen if $(G,*)$ is finite.
4. Let $(S, \circ)$ be a semigroup. If for $x, y \in S, x^2y = y = yx^2$, prove that $(S, \circ)$ is an abelian group. Also prove that in a groupoid $(Z, -)$, there is no left identity bot 0 is a right identity.
5. Show that the set of complex numbers $a + ib\ (a, b \in R)$ for $a^2 + b^2 = 1$ is a group under multiplication. Does it an abelian group? What do you mean by integral power of an element? If x be an element of a group and order of x is 20, find order of x^{15}.
6. Give an example of a finite group whose each element other than identity has same order and also the order of the group is not a prime number. Further prove or disprove the statement "union and intersection of two subgroups of a group is a group."
7. Prove that a finite semigroup where both cancellation laws hold is a group. Does this theorem hold if the semigroup is finite? Justify your answer.
8. Define injective and surjective mapping. A function $f: Z^* \to Z$ is defined by $f(n) = \frac{n}{2}, n\ is\ an\ even\ integer$ and $f(n) = \frac{-n+1}{2}, n\ is\ an\ odd\ integer$. Is f injective, surjective or bijective. [Z^* is the set of all non negative integers].

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