Ramsaday College, Amta

Department of Mathematics

Sub: Abstract & Linear Algebra AND Vector Calculus I

B.Sc.(Honours)-1st yr. **Fourth & Final Class Test: 2018**

F.M. : 25 Marks Time : 1 Hour

(02/04/2018)

Group A (5 marks)

Answer <u>any 1</u> question (5×1=5 Marks)

1. (I) Let $(G,*)$ be a group and $c \in G$. Define a binary composition o on G by $aob = a * c * b$ for all $a, b \in G$. Show that (G, o) is a group and find the identity element of this group.
   (ii) If each element of a group G be its own inverse then prove that G is abelian. Is the converse true? 3+2
2. (i) Let $(G,*)$ be a group. A relation ρ on G is defined by "$a\rho b$" iff $b = gag^{-1}$ for some $g \in G$; $a, b \in G$. Prove that, ρ is an equivalence relation.
   (ii) Let a, b, c be three elements of a group G. Find an element x of Gsuch that $(axb)^{-1} = c^{-1}b^{-1}$. Is such element x unique? 3+2

Group B (20 marks)

Answer <u>any 4</u> questions (5×4=20 Marks)

3. (i) Find the directional derivative of the function $f(x, y, z) = x^2 - y^2 + z^2$ at the point P(1,2,-3) in the direction of the vector PQ, where Q is the point (3,1,2).
   (ii) A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$. Find the components of velocity and acceleration at the time $t = 1$, in the direction of $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$. 3+2
4. Prove that $\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla}.\vec{F}) - \vec{\nabla}^2\vec{F}$. 5
5. For the vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
   (i) Find $grad(log|\vec{r}|)$,
   (ii) Prove that $\frac{\vec{r}}{|\vec{r}|^3}$ is both solenoidal and irrotational. 2+3
6. Prove that the intersection of two subspaces of a vector space V, is a subspace of V; but their union is not, in general, a subspace of V. 3+2
7. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and hence find A^{-1} and A^{50}. 3+2
8. Examine the stability of the following system of equations and solve, if possible :

$$x + 2y + z - 3w = 1$$
$$2x + 4y + 3z + w = 3$$
$$3x + 6y + 4z - 2w = 5.$$

9. Reduce the quadratic form $5x^2+y^2+10z^2-4yz-10xz$ to the normal form. Find its rank, index, signature. Show that it is positive definite. 2+2+1