SEM III

Physics Honours

Paper - CC7

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NUCLEAR STRUCTURE & GENERAL PROPERTIES OF NUCLEI

Nucleus
↓
composed of two different types of elementary particles,
PROTONS & NEUTRONS

Collectively referred to as NUCLEONS

Any particular type of nucleus, a species of nucleus, is called a nuclide.

PROTONS : Nucleus of the lightest and the commonest isotope of hydrogen, a hydrogen atom from which single orbital electron has been removed.

Charge → $+e$

Mass → $1836\ m_e$ $[1836 \times 9.1 \times 10^{-31}\ kg]$
$= 938.3$ MeV

NEUTRONS : Possesses no charge, electrically neutral & hence the name. Mass almost equal to but slightly more than the proton mass

Mass → 939.6 MeV

According to Coulomb's law, the positively charged protons, closely spaced within the nucleus, should repel each other strongly and they should fly apart. It is therefore difficult to explain the stability of the nucleus unless one assumes that nucleons (protons & neutrons) are held together under the influence of very short range attractive force. This force is different from commonly known forces like gravitational or electrical, and is classified as Strong interactions.

ATOMIC NUMBER : No. of protons in the nucleus.
Also called Z-value or proton number.

MASS NUMBER : Sum of no. of protons (Z) and neutrons (N) inside the nucleus.

$$A = N + Z$$

A nucleus of an atom X of atomic number Z and mass no. A that is a nuclide is symbolically represented by ${}^{A}_{Z}X$.

ISOTOPES ↓ Nuclei with same Z, but different A.

Ex: $^{6}_{3}Li$ & $^{7}_{3}Li$

ISOBARS ↓ Nuclei with same A, but different Z.

$^{16}_{8}O$ & $^{16}_{7}N$

ISOTONES ↓ Nuclei with same no. of neutrons

$^{23}_{11}Na$ & $^{24}_{12}Mg$

MIRROR NUCLEI : The pairs of isobaric nuclei wherein the proton no. Z & neutron no. N are interchanged and differ by one unit.

Ex: $^{11}_{6}C$ & $^{11}_{5}B$

$Z = 6$ $Z = 5$

$N = 5$ $N = 6$

NUCLEAR MASS & BINDING ENERGY

Nuclear mass is obtained from the atomic mass $M(A,Z)$ by subtracting the masses of Z orbital electrons.

$$\therefore \quad M_{nuc} = M(A,Z) - Zm_e$$

The above expression is not exact in that the binding energies of the electrons have not been taken into consideration. The error however is small.

The nuclei are very strongly bound & energies of ~few MeV needed to break away a nucleon from the nucleus. In contrast, only a few eV is necessary to detach an orbital e^- from an atom. So, to break up a nucleus of Z protons & N neutrons completely into separate particles, a minimum energy is to be supplied to the nucleus. This energy is called BINDING ENERGY E_B of the nucleus.

Conversely, to build up out of Z protons & N neutrons at rest and separate from one another, a nucleus of mass number A and nuclear charge Z, an amount of energy equal to E_B will be evolved.

But what is the source of this energy?

According to mass-energy equivalence of Special Relativity, the energy equivalent corresponding to complete conversion of a mass m into energy is mc^2 $(E=mc^2)$, where c is the vel. in free space. In forming a nucleus out of the constituent particles, a fraction of the total mass of the constituents disappears and the evolution of energy equal to E_B takes place.

If ΔM be the amount of mass disappeared, then the binding energy $E_B = \Delta M c^2$

If M_H, M_n be the masses of Hydrogen atom & neutron respectively,

$$\Delta M = ZM_H + NM_n - M(A,Z)$$
$$= [ZM_p + NM_n + Zm_e - M_{nuc} - Zm_e]c^2$$
$$= [ZM_p + NM_n - M_{nuc}]c^2$$

UNIT OF ATOMIC MASS → Is defined to be 1/12 th of mass of atom of Carbon isotope ^{12}C taken to be exactly 12 units, symbolised by u, called Unified atomic mass unit. This unit of atomic mass has been in use since 1961 by both physicists and chemists by International agreement. Prior to 1961, the atomic mass units used by physicists and chemists were different. The physicists' unit was previously taken to be one-sixteenth of the mass of ^{16}O isotope (taken to be exactly 16 units) and was called the atomic mass unit (amu). The conversion factor from one scale to the other is given by

$$1\ u : 1\ amu = 1.0003172 : 1$$

The atomic mass unit previously used by the chemists, on the other hand, was one-sixteenth of the average atomic weight of natural oxygen consisting of the three isotopes ^{16}O, ^{17}O and ^{18}O having the relative abundances 99.76%, 0.04% and 0.20% respectively.

To obtain the value of the unit of atomic mass in ^{12}C scale, we note that 1 mole of ^{12}C has the mass of 12 g or 12×10^{-3} kg. Since 1 mole contains N_A atoms, where $N_A = 6.02205 \times 10^{23}$ is the Avogadro number, the mass of each ^{12}C atom is,

$$\frac{12 \times 10^{-3}}{N_A} \quad \text{or} \quad 12 \times 1.660566 \times 10^{-27}\ \text{kg}$$

Hence the unit of atomic mass in ^{12}C scale is

$$1u = \frac{1}{12} \times \frac{12 \times 10^{-3}}{N_A} = 1.660566 \times 10^{-27}\ \text{kg}$$

The energy-equivalent of this amount of mass is

$$1\ u = 1.660566 \times 10^{-27} \times c^2$$
$$= 1.660566 \times 10^{-27} \times 8.98755 \times 10^{16}$$
$$= 14.924427 \times 10^{-11}\ \text{J}$$
$$= \frac{14.924427 \times 10^{-11}}{1.60219 \times 10^{-13}} = 931.502\ \text{MeV}$$

NOTE REGARDING E_B

If $E_B > 0$ i.e. positive, the nucleus is stable and energy from outside is to be supplied to disrupt the nucleus into its constituents separately.

If $E_B < 0$ i.e. negative, the nucleus is unstable and will disintegrate by itself.

The E_B- value is a measure of the stability of the nucleus. More the E_B, more is the stability.

MASS DEFECT

Difference b/w measured atomic mass M(A, Z) expressed in u, and the mass number A of a nuclide is called mass defect $\Delta M'$.

$$\Delta M' = M(A,Z) - A$$

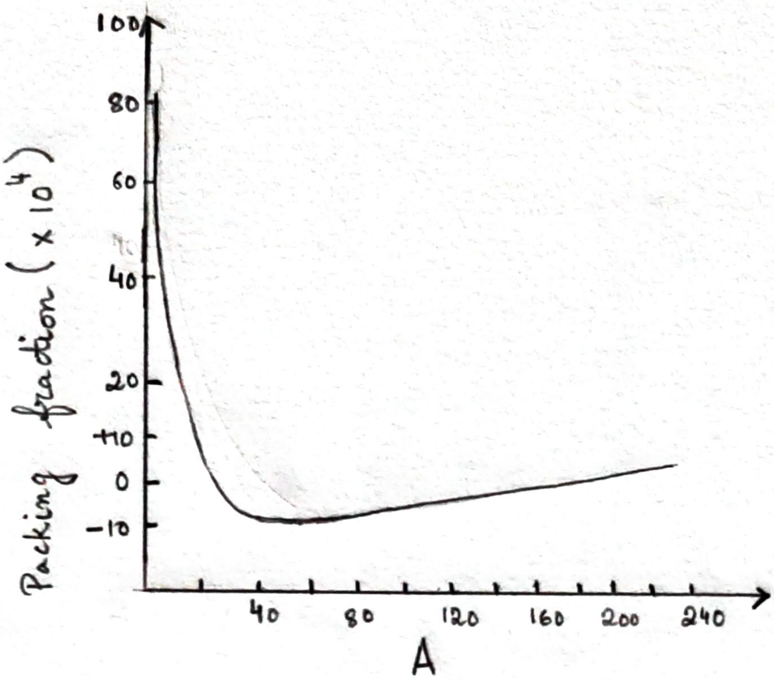
PACKING FRACTION

Defined as mass defect per nucleon in the nucleus.

$$f = \frac{\Delta M'}{A} = \frac{M(A,Z) - A}{A}$$

Or, $M(A,Z) = A(1+f)$

PACKING FRACTION CURVE



It is found that the packing fraction f varies in a systematic manner with the mass number A. The nature of the variation is shown graphically in Fig 1.

From the figure it is seen that for very light nuclei the packing fraction is positive and decreases rapidly with increasing A. It becomes negative for A greater than about 20, attains a minimum (negative) at $A \sim 60$. It then rises slowly for higher A and becomes positive again for A greater than about 180.

This systematic variation of f with A can be understood from nuclear binding energy considerations.

If the binding energy E_B of a nucleus ${}^{A}_{Z}X$ is divided by the mass number A, we get the binding energy per nucleon in the nucleus, which is known as the binding fraction (f_B) and is given by

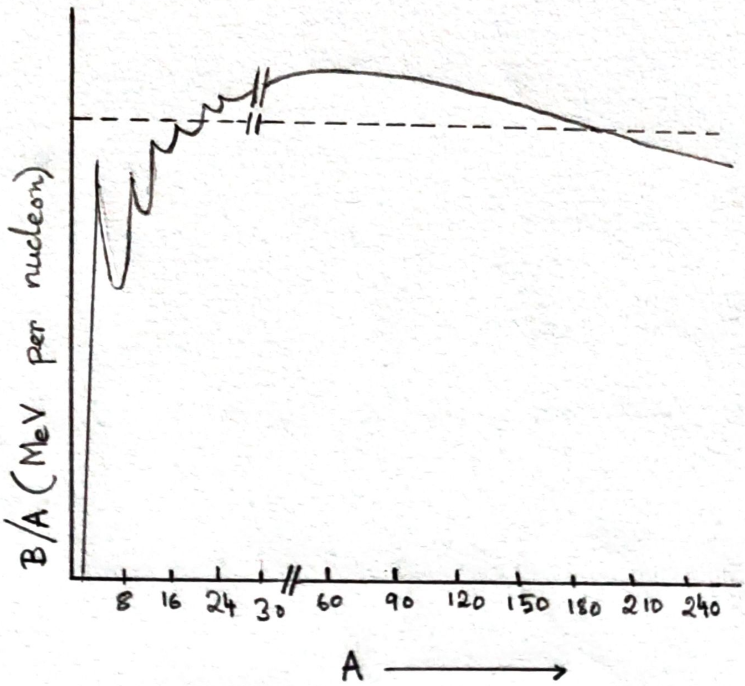
$$f_B = \frac{E_B}{A} = \frac{ZM_H + NM_n - M(A,Z)}{A}$$

Here we have assumed that the masses are expressed in energy unit so that c^2 on r.h.s. of above equation has been omitted.

BINDING FRACTION VS. MASS NUMBER CURVE

IMPORTANT POINTS :-

→ f_B is very small for very light nuclei & goes on increasing rapidly with increasing A and reaches a value ~ 8 MeV/nucleon for the mass number A ~ 20. Thereafter, the rise of the curve is much slower, reaching a maximum value of 8.7 MeV per nucleon for A = 56. If A is increased still further, the curve decreases slowly.



BINDING FRACTION CURVE

→ The variation in f_B is very slight in the mass number range $20 < A < 180$ and in this region f_B may be considered to be virtually constant with a mean value ~ 8.5 MeV/nucleon.

→ For $A > 180$, that is for very heavy nuclei, f_B decreases monotonically with increasing A.

→ A rapid fluctuation in f_B is noted for very light nuclei with peaks in the curve of this region, corresponding to even-even nuclei, such as ^4He, ^8Be, ^{12}C, ^{16}O i.e. with mass number $A = 4n$, where $n = 1, 2, 3, \ldots$. Peaks in the curve are also seen at Z or N equal to 20, 28, 50, 82, 126. These are called Magic Numbers.

The significance of the peaks is that the corresponding nuclei are more stable relative to those in the neighbourhood.

Complementarity of binding fraction curve & packing fraction curve

The binding fraction curve is complementary to the packing fraction curve.

$$E_B = ZM_H + NM_n - M(A,Z)$$
$$= Z(1+f_H) + N(1+f_n) - A(1+f)$$
$$= Z + N + Zf_H + Nf_n - A - Af$$
$$= Zf_H + Nf_n - Af \quad (\because A = Z+N)$$

$$f_B = \frac{E_B}{A} = \underbrace{\frac{Zf_H + Nf_n}{A}}_{\text{Nearly constant}} - f$$

Thus, f_B increases or decreases as f decreases or increases.