Ramsaday College, Amta

Class Test Examination 2018

Subject: Mathematics (Sequence & Integration)

Full Marks: 25 **Time: 60 minutes**

Date: 23/03/2018

Group-A

Answer any three questions: (5×3=15)

1. i) A sequence $\{x_n\}$ of real numbers is defined by the recurrence relation $x_{n+1} = x_n(2 - x_n) \forall n \in N$, 0$< x_1 < 1$. Show that the sequence is convergent. Find its limit.

ii) Verify Bolzano-Weierstrass theorem for the sequence $\{u_n\}$ given by $u_n = sin\frac{n\pi}{2}$. (3+2)

2. i) Find upper and lower limits of the sequence $\{(cos\frac{n\pi}{4})^{(-1)^n}\}$.

ii) Prove or disprove: Product of a divergent sequence and a null sequence is a null sequence. (3+2)

3. When a sequence $\{u_n\}$ is called Cauchy sequence? If $\{u_n\}$ and $\{v_n\}$ are Cauchy sequence then prove that $\{u_n v_n\}$ is a Cauchy sequence. (1+4)

4. i) Prove that a bounded sequence is convergent iff it has only one sub- sequential limit.

ii) Prove or disprove: Every bounded sequence is Cauchy sequence. (4+1)

(P.T.O)

5. i) Define Cantor's theorem on nested intervals. When the intervals are not closed, is this theorem true? Justify your answer.

ii) Prove that $\lim_{n\to\infty} \frac{1+(2)^{\frac{1}{2}}+(3)^{\frac{1}{3}}+\cdots\ldots+(n)^{\frac{1}{n}}}{n}$ =1. (3+2)

Group-B

Answer any two questions: (5×2=10)

6. If $I_n = \int_0^1 x^n \tan^{-1}x \, dx$, prove that for n>2, $(n+1)I_n + (n-1)I_{n-1} = \frac{\pi}{2} - \frac{1}{n}$.

7. Evaluate : $\lim_{n\to\infty}[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \cdots .. + \frac{1}{n}]$.

8. Find $\int \frac{dx}{(\cos\alpha+\cos x)^2}$.

9. Prove that $\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)(n-3)\ldots.4.2}{n(n-2)\ldots.5.3.1}$, if n be any odd positive integer and n$>$ 1.

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