

# Ramsaday College

Class: B.Sc. Part-I

Full Marks: 100

Subject: Physics (Hons.)

Paper-First

Model Question Paper: 2018

Answer Question *No. 1* and any four questions each from *Unit-1* and *Unit-2*

1. Answer any ten questions:

- (a) Show that the infinite series  $\sum \frac{1}{n^p}$  is convergent if  $p > 1$ .
- (b) Show that a Hermitian matrix remain Hermitian under unitary transformation.
- (c) What is the Fourier transform of  $\delta(x)$ ?
- (d) A progressive harmonic wave is represented as  $y(x,t) = a \sin(0.5x - 10t)$  where  $x$  is in meters and  $t$  is in seconds. Obtain the wave velocity.
- (e) Establish the relation  $I_c = \beta E_B + (1 + \beta) I_{CBO}$  for the transistor in active CE mode.
- (f) Evaluate  $\iint_R xy \, dy$  where  $R$  is the quadrant of the circle with  $x \geq 0, y \geq 0$ .
- (g) Define bandwidth and quality factor with respect to resonance.
- (h) Determine the condition under which the differential equation  $C_1 \frac{\partial \psi}{\partial t} + C_2 \frac{\partial^2 \psi}{\partial t^2} + V(x,t)\psi(x,t) = 0$ , can be solved using the method of separation of variables.
- (i) The band gap of GaAs is 1.98 eV. Determine the wavelength of e.m. radiation upon recombination of holes and electrons.
- (j) A surface in 3-dimensions is described by  $u(x,y,z)=C$  where  $C$  is a constant. Show that
- (k) Verify the Boolean identity  $(A+B)(B+C)(C+A)=AB+BC+CA$
- (l) State the order and degree of the differential equation  $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0$ .

Unit-1

2. (a) Express the divergence operator I spherical polar coordinates starting from the expression of the same in Cartesian coordinates.  
 (b) Find the first four terms in the Maclaurin series expansion of  $(1+x)\ln(1+x)$ .  
 (c) Six coins are tossed simultaneously. What is the probability of (i) 2 heads (ii) at least 2 heads.

$$4+2+(2+2)$$

3. (a) Apply Frobenius method to the equation  $\frac{d^2y}{dx^2} + \omega^2 y = 0$  setting  $y(x) = \sum a_\lambda x^{k+\lambda}$

with  $a_0 \neq 0$ .

- (i) Verify that the indicial equation is  $k(k-1)=0$ .  
 (ii) For  $k=1$ , show that  $a_1$  is necessarily zero.  
 (iii) Find the recurrence relation.  
 (iv) Using the recurrence relation, show that  $k=1$  leads to the solution

$$y(x) = \frac{a_0}{\omega} \sin \omega x.$$

- (b) Legendre polynomials  $P_n(x)$  may be expressed as  $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$ . Using

this show that  $P_n(-x) = (-1)^n P_n(x)$ . (2+2+2+2)+2

4. (a) Find the eigenvalues of and normalized eigenvectors of the matrix  $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ .

(b) If a matrix is both Hermitian and unitary, show that all its eigen values are  $\pm 1$ .

(c) Show that the product of two symmetric matrices is symmetric only the commute.

$$(2+3)+3+2$$

5. (a) Show that  $\frac{1}{3} \oint_S \vec{r} \cdot d\vec{s} = V$  where V is the volume enclosed by the surface S.

(b) By the Stokes theorem proved that  $\vec{\nabla} \times \vec{\nabla} \phi = 0$ .

(c) A particle moves along the curve  $x = 2t^2$ ,  $y = 4t^2 - 1$ ,  $z = 5t - 3$ , where  $t$  denotes time.

Find the component of acceleration at time  $t=1$  in the direction  $(\hat{i} - 2\hat{j} + 2\hat{k})$ .

(d) Show that the line integral of  $\vec{F} = -\hat{i}y + \hat{j}x$  around a continuous closed curve in  $x$ - $y$  plane is twice the area enclosed by the curve. 2+2+3+3

6. (a) Solve  $x^2 \frac{d^2y}{dx^2} - 6y = 0$  by method of Frobenius.

(b) Solving the question using the method separation of variables,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \text{ With } V=0 \text{ for } Y=0 \text{ and } Y=\pi$$

$$V=V_0 \text{ for } x=1$$

$$V(-x,y)=V(x,y) \span style="float: right;">4+6$$

7. (a) Find the Fourier transform  $F(k)$  of the function  $f(x)=N e^{-\alpha x^2}$  where  $N$  and  $\alpha$  are constants. Calculate the standard deviation of both  $f(x)$  and  $F(k)$ . Plot  $F(k)$  against  $k$  and  $f(x)$  against  $x$ .

(b) Plot the function  $f(x)$  defined by  $f(x)=0$  for  $-\pi < x < 0$ ,  $f(x)=1$  for  $0 < x < \pi$ ,  $f(x+\pi)=f(x)$  and find the Fourier series expansion for it.

(c) Evaluate the  $\int_{-1}^{+1} e^x \delta'(x) dx$ , where the prime denotes differentiation with respect to  $x$ .

(2+1+1)+4+2

### Unit-2

8. (a) Define principal points and nodal points of an optical system.

(b) Find out the condition for smallest separation between an object and its real image produced by a converging lens.

(c) State Fermat's principle. Use it to establish conjugate foci relations for refraction at a spherical surface.

3+3+(1+3)

9. (a) Solved the equation of motion of a damped forced SHM and derived the condition for velocity resonance.
- (b) Contract the differential equation of wave motion starting from the equation of a plane progressive harmonic wave given by  $y(x,t) = A \sin\left(\omega t - \frac{\omega}{v}x + \alpha\right)$  where the notations have their as usual meanings.
- (c) Distinguish between amplitude resonance and velocity resonance.

(4+2)+2+2

10. (a) Draw a two-input positive logic diode AND circuit and explained its operation.
- (b) Design a two-input XOR gate using NOR gates excursively.
- (c) Verify the following identities:

(i)  $(A + B)(A + C) = A + BC$

(ii)  $(A + \bar{A}B) = A + B$

(1+2)+3+(2+2)

11. (a) State and prove the maximum power transfer theorem.
- (b) Find out the value of  $R_L$  such that maximum is delivered to it. Also, find the value of maximum power.
12. (a) Explained the behavior of a p-n junction diode under reverse bias and hence explain the characteristics of a p-n junction diode.
- (b) Sketch the output characteristics of a CE mode n-p-n transistor and explained it operation.
- (c) Sketch the basic structure of an n-channel depletion MOSFET.

(3+1)+(2+2)+2

13. (a) Define linear and angular magnification of a optical system. Derive Helmholtz-Legrange relation between these two types of magnification.
- (b) Define paraxial ray. Derive the system matrix of two thin lenses separated by a distance 't' in air.

(1+1+4)+1+3

